## Grade 10 Assessment Standards

### Learning Outcome 1: Number and Number Relationships

*When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.*

**Grade 10**
We know this when the learner is able to:

**10.1.1**
Identify rational numbers and convert between terminating or recurring decimals and the form:

**10.1.2**
(a) Simplify expressions using the laws of exponents for integral exponents.
(b) Establish between which two integers and simple surd lies.
(c) Round rational and irrational numbers to an appropriate degree of accuracy

**10.1.3**
Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore linear) and hence:

(a) Make conjectures and generalisations.
(b) Provide explanations and justifications and attempt to prove conjectures.

$$\frac{a}{b}; a,b \in Z; b \neq 0$$

**10.1.4**
Use simple and compound growth formulae: $A = P(1+ni)$ and $A = P(1+i)^n$ to solve problems including; interest, hire-purchase, inflation, population growth and other real-life problems.

**10.1.5**
Demonstrate an understanding of the implications of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel).

**10.1.6**
Solve non-routine, unseen problems.

## Grade 11 Assessment Standards

### Learning Outcome 1: Number and Number Relationships

*When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.*

We know this when the learner is able to:

**11.1.1**
Understand that not all numbers are real. (This requires the recognition but not the study of non-real numbers.)

**11.1.2**
(a) Simplify expressions using the laws of exponents for rational exponents.
(b) Add, subtract, multiply and divide simple surds:

(e.g. $\sqrt{3} + \sqrt{12} = 3\sqrt{3}$ and $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

(c) Demonstrate an understanding of error margins.

**11.1.3**
Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence:
(a) make conjectures and generalisations
(b) provide explanations and justifications and attempt to prove conjectures.

**11.1.4**
Use simple and compound decay formulae; $A = P(1-ni)$ and $A = P(1-i)^n$ to solve problems including; straight line depreciation and depreciation on a reducing balance).

**11.1.5**
Demonstrate an understanding of different periods of compounding growth and decay (including effective compounding growth and decay and including effective and nominal interest rates).

**11.1.6**
Solve non-routine, unseen problems.

## Grade 12 Assessment Standards

### Learning Outcome 1: Number and Number Relationships

*When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.*

We know this when the learner is able to:

**12.1.1**
Not applicable.

**12.1.2**
Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems (e.g. growth and decay se 12.1.4(a))

**12.1.3**
(a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series.
(b) Correctly interpret sigma notation.
(c) Prove and correctly select the formula for and calculate the sum of series, including:

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} a \times r^{i-1} = \frac{a(r^n - 1)}{r-1}; r \neq 1$$ for -1 < $r$ < 1

(d) Correctly interpret recursive formulae: e.g.

$$T_{n+1} = T_n + T_{n-1}$$

**12.1.4**
(a) Calculate the value of in the formula: $A = P(1 \pm i)^n$
(b) Apply knowledge of geometric series to solving annuity, bond repayments and sinking fund problems, with or without the use of the formulae:

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$$

**12.1.5**
Critically analyse investment and loan options and make informed decisions as to the best option(s) (including pyramid and micro-lenders' schemes).

**12.1.6**
Solve non-routine, unseen problems.

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### Learning Outcome 2: Functions and Algebra (Grade 10)

*The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.*

We know this when the learner is able to:

**10.2.1**
(a) Demonstrate the ability to work with various types of functions, including those listed in the following Assessment Standard.
(b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibility between these representations (tables, graphs, words and formulae).

**10.2.2**
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence to generalise the effects of the parameters a, and q on the graphs of functions including:

$$y = ax + q$$
$$y = ax^2 + q$$
$$y = \frac{a}{x} + q$$
$$y = ab^x + q; b > 0$$
$$y = a\sin(x) + q$$
$$y = a\cos(x) + q$$
$$y = a\tan(x) + q$$

### Learning Outcome 2: Functions and Algebra (Grade 11)

*The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.*

We know this when the learner is able to:

**11.2.1**
(a) Demonstrate the ability to work with various types of functions, including those listed in the following Assessment Standard.
(b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibility between these representations (tables, graphs, words and formulae).

**11.2.2**
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence to generalise the effects of the parameters $a$, and $q$ on the graphs of functions including:

$$y = \sin(kx) \qquad y = \sin(x+p)$$
$$y = \cos(kx) \qquad y = \cos(x+p)$$
$$y = \tan(kx) \qquad y = \tan(x+p)$$
$$y = a(x+p)^2 + q$$
$$y = \frac{a}{x+p} + q$$
$$y = ab^{x+p} + q; b > 0$$

### Learning Outcome 2: Functions and Algebra (Grade 12)

*The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.*

We know this when the learner is able to:

**12.2.1**
(a) Demonstrate the ability to work with various types of functions, including those listed in the following Assessment Standard.
(b) Demonstrate knowledge of the formal definition of a function.

**12.2.2**

(a) Investigate and generate graphs of the inverse relations of functions, in particular the inverses of:

$$y = ax + q$$
$$y = ax^2$$
$$y = a^x; a > 0$$

(b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function.

**10.2.3**
Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed in 10.2.2 above:
(a) domain and range;
(b) intercepts with the axes;
(c) turning points, minima and maxima;
(d) asymptotes;
(e) shape and symmetry;
(f) periodicity and amplitude;
(g) average gradient (average rate of change);
(h) intervals on which the function increases or decreases;
(i) the discrete or continuous nature of the graph.

**10.2.4**
Manipulate algebraic expressions:
(a) multiplying a binomial by a trinomial;
(b) factorising trinomials;
(c) factorising by groups in pairs;
(d) simplifying algebraic fractions with monomial denominators.

**10.2.5**
Solve:
(a) linear equations;
(b) quadratic equations by factorisation;
(c) exponential equations of the form $ka^{x+p} = m$ (including examples solved by trial and error);
(d) linear inequalities in two variables simultaneously (numerically, algebraically and graphically).

**10.2.6**
Use mathematical models to investigate problems that arise in real-life contexts:
(a) making conjectures, demonstrating and explaining their validity;
(b) expressing and justifying mathematical generalisations of situations;
(c) using the various representations to interpolate and extrapolate;
(d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and features. (Examples should include issues related to health, social, economic, cultural, political and environmental matters).

**10.2.7**
Investigate the average rate of change of a function between two values of the independent variable, demonstrating an intuitive understanding of average rate of change over different intervals (e.g. investigate water consumption by calculating the average rate of change over different time intervals and compare results with the graph of the relationship).

**11.2.3**
Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed in 11.2.2 above:
(a) domain and range;
(b) intercepts with the axes;
(c) turning points, minima and maxima;
(d) asymptotes;
(e) shape and symmetry;
(f) periodicity and amplitude;
(g) average gradient (average rate of change);
(h) intervals on which the function increases or decreases;
(i) the discrete or continuous nature of the graph.

**11.2.4**
Manipulate algebraic expressions:
(a) By completing the square;
(b) simplifying algebraic fractions with binomial denominators.

**11.2.5**
Solve:
(a) quadratic equations (by factorisation, by completing the square, and by using the quadratic formula) and quadratic inequalities in one variable and interpret the solution graphically;
(b) equations in two unknowns, one of which is linear and one of which is quadratic, algebraically and graphically.

**11.2.6**
Use mathematical models to investigate problems that arise in real-life contexts:
(a) making conjectures, demonstrating and explaining their validity;
(b) expressing and justifying mathematical generalisations of situations;
(c) using the various representations to interpolate and extrapolate;
(d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and features. (Examples should include issues related to health, social, economic, cultural, political and environmental matters).

**11.2.7**
Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of curve at a point.

**11.2.8**
(a) Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by numerical search along the boundary of feasible region.
(b) Solve a system of linear equations to find the co-ordinates of the vertices of the feasible region.

**12.2.3**
Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of the inverses of the functions listed in 12.2.2 above:
(a) domain and range;
(b) intercepts with the axes;
(c) turning points, minima and maxima;
(d) asymptotes;
(e) shape and symmetry;
(f) average gradient (average rate of change);
(g) intervals on which the function increases or decreases;

**12.2.4**
Factorise third degree polynomials (including examples which require the factor theorem).

**2.2.5**
Not applicable.

**12.2.6**
Not applicable.

**12.2.7**
(a) Investigate and use instantaneous rate of change of a variable when interpreting models of situations:
- demonstrate an intuitive understanding of the limit concept in the context of approximating the rate of change or gradient of a function at a point;
- establishing the derivatives of the following functions from first principles.

$$f(x) = b$$
$$f(x) = x$$
$$f(x) = x^2$$
$$f(x) = x^3$$
$$f(x) = \frac{1}{x}$$

and then generalise to the derivative of:

$$f(x) = x^n$$

(b) Use the following rules of differentiation:

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$$
$$\frac{d}{dx}\left[k.f(x)\right] = k\frac{d}{dx}\left[f(x)\right]$$

(c) Determine the equations of tangents to graphs.
(d) Generate sketch graphs of cubic functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the $x$-intercepts.
(e) Solve practical problems involving optimisation and rates of change.

**12.2.8**
Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.

## Learning Outcome 3: Space, Shape and Measurement

*The learner is able to recognise, describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.*

We know this when the learner is able to:

**10.3.1**
Understand and determine the effect on the volume and surface area of right prisms and cylinders, of multiplying any dimension by a constant factor $k$.

**10.3.2**
(a) Through investigations, produce conjectures and generalisations related to triangles, quadrilaterals and other polygons, and attempt to validate, justify, explain or prove them using any logical method (Euclidean, co-ordinate and/or transformation).
(b) Disprove false conjectures by producing counter-examples.
(c) Investigate alternative definitions of various polygons (including the isosceles, equilateral and right-angled triangle, the kite, parallelogram, rectangle, rhombus and square).

**10.3.3**
Represent geometric figures on a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1; y_1)$ and $(x_2; y_2)$, a formula for calculating:

(a) the distance between the two points;
(b) the co-ordinates of the mid-point of the line segment joining the points;
(c) the gradient of the line segment joining the points.

**10.3.4**
Investigate, generalise and apply the effect of the following transformations of the point $(x; y)$:

(a) a translation of $p$ units horizontally and $q$ units vertically;
(b) a reflection in the $x$-axis, the $y$-axis or the line $y = x$.

**10.3.5**
Understand that the similarity of triangles is fundamental to the trigonometric functions $\sin\theta$, $\cos\theta$ and $\tan\theta$, and is able to define and use the functions.

**10.3.6**
Solve problems in two dimensions by using the trigonometric functions ($\sin\theta$, $\cos\theta$ and $\tan\theta$) in right-angled triangles and by constructing and interpreting geometric and trigonometric models (examples to include scale drawings, maps and building plans).

**10.3.7**
Demonstrate an appreciation of the contributions to the history of the development and use of geometry and trigonometry by various cultures through a project.

## Learning Outcome 3: Space, Shape and Measurement

*The learner is able to recognise, describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.*

We know this when the learner is able to:

**11.3.1**
Use the formulae for surface area and volume of right pyramids, right cones, spheres and combinations of these geometric objects.

**11.3.2**
(a) Investigate necessary and sufficient conditions for polygons to be similar.
(b) Prove and use (accepting results established in earlier grades):
- that a line drawn parallel to one side of a triangle divides the other two sides proportionately (the Mid-point Theorem as a special case of this theorem)
- that equiangular triangles are similar
- that triangles with sides in proportion are similar;
- the Pythagorean Theorem by similar triangles.

**11.3.3**
Use a Cartesian co-ordinate system to derive and apply:
(a) the equation of a line through two given points;
(b) the equation of a line through one point and parallel or perpendicular to a given line;
(c) the inclination of a line.

**11.3.4**
Investigate, generalise and apply the effect on the co-ordinates of:
(a) the point $(x; y)$ after rotation around the origin through an angle of $90^\circ$ or $180^\circ$;
(b) the vertices $(x_1; y_1)$, $(x_2; y_2)$,…$(x_n; y_n)$ of the polygon after enlargement through the origin, by a constant factor $k$.

**11.3.5**
(a) Derive and use the values of the trigonometric functions (in surd form where applicable) of $30^\circ$, $45^\circ$, and $60^\circ$.
(b) Derive and use the following identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta = 1$$

(c) Derive the reduction formulae for:

$$\sin(90^\circ \pm \theta) \quad \cos(90^\circ \pm \theta)$$
$$\sin(180^\circ \pm \theta) \quad \cos(180^\circ \pm \theta) \quad \tan(180^\circ \pm \theta)$$
$$\sin(360^\circ \pm \theta) \quad \cos(360^\circ \pm \theta) \quad \tan(360^\circ \pm \theta)$$
$$\sin(-\theta) \quad \cos(-\theta) \quad \tan(-\theta)$$

(d) Determine the general solution of trigonometric equations.
(e) Establish and apply the sine, cosine and area rules.

**11.3.6**
Solve problems in two dimensions by using the sine, cosine and area rules; and by constructing and interpreting geometric and trigonometric models.

**11.3.7**
Demonstrate an appreciation of the contributions to the history of the development and use of geometry and trigonometry by various cultures through educative forms of assessment. (e.g. an investigative project).

## Learning Outcome 3: Space, Shape and Measurement

*The learner is able to recognise, describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.*

We know this when the learner is able to:

**12.3.1**
Not applicable.

**12.3.2**
(a) Accept the following axioms:
- results established in earlier grades;
- a tangent is perpendicular to the radius, drawn at the point of contact with the circle, and then investigate and prove the theorems of the geometry of circles:
- the line drawn from the centre of the circle, perpendicular to a chord, bisects the chord and its converse;
- the perpendicular bisector of a chord passes through the centre of the circle; the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle;
- angles subtended by a chord at the circle on the same side of the chord are equal and its converse;
- the opposite angles of a cyclic quadrilateral are supplementary and its converse;
- two tangents drawn to a circle from the same point outside the circle are equal in length.
- the angles between a tangent and a chord, drawn to the point of contact of the chord, are equal to the angles which the chord subtends in the alternate chord segments and its converse.

(b) Use the theorems listed above to:
- make and prove or disprove conjectures;
- prove riders.

**12.3.3**
Use a two dimensional Cartesian co-ordinate system to derive and apply:
(a) the equation of a circle (any centre);
(b) the equation of a tangent to a circle given a point on the circle.

**12.3.4**
(a) Use the compound angle identities to generalise the effect on the co-ordinates of the point $(x; y)$ after rotation about the origin through an angle $\theta$.
(b) Demonstrate the knowledge that rigid transformations (translations, reflections, rotations and glide reflections) preserve shape and size, and that the enlargement preserve shape but not size.

**12.3.5**
Derive and use the following compound angle identities:

(a) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

(b) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

(c) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

(d) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

**12.3.6**
Solve problems in two and three dimensions by constructing and interpreting geometric and trigonometric models.

**12.3.7**
Demonstrate a basic understanding of the development and uses of geometry through history and some familiarity with other geometries (e.g. spherical geometry, taxi-cab geometry, and fractals).

**Learning Outcome 4: Data Handling and Probability**

*The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.*

We know this when the learner is able to:

**10.4.1**
(a) Collect, organise and interpret univariate numerical data in order to determine:
- Collect, organise and interpret univariate numerical data in order to determine:
- measures of dispersion: range, percentiles, quartiles, interquartile and semi-inter-quartile range.

(b) Represent data effectively, choosing appropriately from:
- bar and compound bar graphs;
- histogram (grouped data);
- frequency polygons;
- pie charts;
- line and broken line graphs.

**10.4.2**
(a) Use probability models for comparing the relative frequency of an outcome with the probability of an outcome (understanding, for example that it takes a very large number of trials before the relative frequency of throwing a head approaches the probability of throwing a head).
(b) Use Venn diagrams as an aid to solving probability problems, appreciating and correctly identifying:

**10.4.3**
(a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).
(b) Effectively communicate conclusions and predictions that can be made from the analysis of data.

**10.4.4**
Not Applicable

**10.4.5**
Use theory learned in this grade in an authentic integrated form of assessment (e.g. in an investigative project).

**Learning Outcome 4: Data Handling and Probability**

*The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.*

We know this when the learner is able to:

**11.4.1**
(a) Calculate and represent measures of central tendency and dispersion in univariate numerical data by:
- five number summary (maximum, minimum and quartiles);
- box and whisker diagrams;
- ogives;
- calculating the variance and standard deviation of sets of data manually (for small sets of data) and using available technology (for larger sets of data), and representing results graphically using histograms and frequency polygons.

(b) Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data (problems should include issues related to health, social, economic, cultural, political and environmental issues).

**11.4.2**
(a) Correctly identify dependent and independent events (e.g. from two-way contingency tables or Venn diagrams) and therefore appreciate when it is appropriate to calculate the probability of two independent events occurring by applying the product rule for independent events: P(A and B) = P(A).P(B).
(b) Use tree and Venn diagrams to solve probability problems (where events are not necessarily independent).

**11.4.3**
(a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).
(b) Effectively communicate conclusions and predictions that can be made from the analysis of data.

**11.4.4**
Differentiate between symmetric and skewed data and make relevant deductions.

**11.4.5**
Use theory learned in this grade in an authentic integrated form of assessment (e.g. in an investigative project).

**Learning Outcome 4: Data Handling and Probability**

*The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.*

We know this when the learner is able to:

**12.4.1**
(a) Demonstrate the ability to draw a suitable sample from a population and understand the importance of sample size in predicting the mean and standard deviation of a population.
(b) Use available technology to calculate the regression function which best fits the given set of bivariate numerical data.
(c) Use available technology to calculate the correlation co-efficient of a set of bivariate numerical data to make relevant deductions.

**12.4.2**
Generalise the fundamental counting principle (successive choices from $m_1$ then $m_2$ then $m_3$ … options create $m_1.m_2.m_3$ … different combined options) and solve problems using the fundamental counting principle.

**12.4.3**
(a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).
(b) Effectively communicate conclusions and predictions that can be made from the analysis of data.

**12.4.4**
Identify data which is normally distributed about a mean by investigating appropriate histograms and frequency polygons.

**12.4.5**
Use theory learned in this grade in an authentic integrated form of assessment (e.g. in an investigative project).